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**Sub optimal
investment in
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**Sub optimal investment in
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Summary : in a context of concentrated electricity industries and entry barriers, governments may worry that incumbent firms strategically under-invest in generation. Associated with the well known short-term strategy of production restriction, suboptimal investment allows firms to increase price and profits, and retain long-term market power. When these strategies include reserve capacity investment, system reliability could be altered. The paper analyses a policy response using a public firm to invest in generating capacity and produce competitively so as to restore the long-term social optimum. A dynamic three-stage game is modelled to analyse the capacity choices in a mixed oligopoly with private leaders and a public follower. The model considers two stages of investment, (the first by the private firms, the second by the public one), and a stage of production to distinguish long-term and short-term market power. It shows that short-term market power of private firms could prevent the public firm from restoring the long-term optimum. Contrary to usual result on commitment, it is the inability of private firms to commit to a given production level that allows them to get strictly positive profits. We establish that for high degree of concentration of the industry and low elastic demand, the private firms are still able to get strictly positive profits. The distance to optimal level of investment may be decreasing with respect to the number of firms.

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1. Introduction

Ensuring enough generation capacity to meet future electricity demand progressively became a contentious issue in the design of the reforms, particularly since some recent crisis. One motivation behind the reforms is to encourage efficient investment and to avoid situations of overcapacity observed in the former regime of regulated monopoly. But there are growing concerns about the ability of the new liberalised market regime to invest sufficiently in building capacity required to avoid long periods of high prices and to ensure supply security.

Three main reasons are highlighted in the literature to explain the sub-optimality of investment in generation. The first refers to market power. Because of the inelastic nature of demand, even a slight shortfall of available capacity or unexpected high loads can provoke a dramatic increase in price. Every generator regardless of their market share can potentially benefit from a sub-optimal investment in generation capacity in the system, which will provoke profitable periods of price spikes. Hence generators are incited to under-invest, instead of aggressively investing for expanding their market shares. This is argued as one of the explanations concerning the lack of generation capacity during the Californian electricity crisis (Borenstein, 2002).

The second reason is the exceptional volatility of electricity price and the corresponding difficulty of risk management, in particular for the development of futures and forward contracts. Such volatility is explained by the short-run inelasticity of supply and demand, and the non-storable nature of electricity: "On a market on which consumers cannot react to prices in a situation of severe capacity tension, there are no limits to the prices that the producers can fix when a shortfall appears" which favours exercise of market power (R. Green, 2001). During the period of tight supply, market has no way to set price and at these times price is controlled by market power, as Stoff (2002) clearly shows. Moreover the price spikes which are necessary to attract investment in "peakers" raise problems of public acceptability of reforms. The main problem which is amplified for the investment in peaking units is that it is not possible to distinguish between scarcity rent and market power effects. Every generator, even the smallest, has the ability to make the prices rise by restricting its production in particular during high loads. Even if large price spikes are efficient signals of scarcity, the public suspicion of market power lead regulators to impose price caps. This adds to the disincentive effects of volatility to discourage investment in peaking units.

This volatility is particularly problematic in two ways. First, there is a risk that markets overreact to recurrent price spikes and over-invests in base load, semi-load and peaking equipment. This results from uncoordinated investment decisions and may result in a dramatic price drop and bankruptcy of new entrants, as observed in US regional markets after 2000 with "boom and bust" cycles. These specific risks could discourage investment in generation by risk averse entrants (Neuhoff et De Vries, 2004; Finon, 2006; Joskow, 2006). Secondly, infra-marginal rents during the peak periods are needed to ensure the profitability of both peak and base load investments, yet these rents which depend on the magnitude of price spike after the commissioning of equipments are highly uncertain.

The third reason of investment deficit which is also evoked in the literature on the reliability of supply and the capacity adequacy is the existence of a public good. This

concerns the investment in “peakers” in order to reach an adequate capacity which includes sufficient reserve capacity to face in real time every random situation on availability of generation equipments and demand during high load, given the non storability of the electricity. Literature concerns specific instruments which aim to finance peak equipments and compensate missing revenues (see for instance Joskow and Tirole, 2004; Crampton and Stoff, 2006; Joskow, 2006; De Vries, 2007).

We analyse here the efficiency of a policy to compensate sub-optimal investment resulting from market power exercise by mandating one public firm to behave in a benevolent way on the short and long term. This benevolent firm considers the oligopolistic behaviour of the other firms when maximising the social surplus. This firm could be a public or a semi-public one, knowing that a number of historic operators remain under this property regime in some European countries and North American regions. Experience shows that public or semi-public firms often play a special role in the short term competition. As they tend to identify part of their objectives to the public interest and the social welfare, they avoid exerting market power to capture high surplus.¹

This approach is relevant to cope with policies aiming to reduce investment deficit in reserve capacity resulting from producers’ risk aversion and market power, by the help of public procurement for long term reserve contracts with the transmission system operator (TSO) or by the TSO’s direct investment in peaking units under the control of the regulator. This capacity mechanism is already used in some European countries (Sweden, Norway, France) and New Zealand within a precise regulatory framework aiming to avoid market distortions (De Vries, 2007). Procurement by the benevolent company, here the TSO mandated by the regulator should compensate the suboptimal investment resulting from the exercise of long term market power in reserve capacity and improve the reliability of the system.

In the present paper, we model a three stage capacity choice game in a mixed oligopoly in the line of the literature of two-stage dynamic oligopoly models around investment along several contributions including von der Fehr and Harbord (1997), Murphy and Smeers (2002) and Boom (2002, 2003). Private firms choose capacities before a benevolent public firm, production quantities are set in a third stage à la Cournot with capacity constraints. The public firm acts as a follower because its intervention is seen as an ex-post remedy to a lack of private investment. Costs are assumed linear and the public firm as efficient as private ones. In that case it seems obvious that the welfare could be maximized by the public firm intervention. One may think that the public firm only has to invest in missing capacities to reach the long term social optimum equalizing price with long term marginal cost. Actually, it may not be possible for the public firm to do so because of the short term market power of private firms. If the public firm invests in order to make the market reach optimal capacity, the private firms restrict their production and there is a public loss due to this restriction. At the first stage of the game, when private firms choose their capacity they could gain strictly positive profit by investing in sufficient capacity to put the public firm in the situation described above.

This work is therefore at the crossing of the mixed oligopoly literature and the ‘commitment game’ literature that are reviewed below. Contrary to the former the

¹ Some examples can be recorded in the Nordic experiences. Public ownership tends to help cooperation between firms and regulators in view of social welfare and to deter rent-seeking by market power exercise on the power exchange (Magnus and Middtun, 2000; von der Fehr, Amundsen and Bergman, 2006).

efficiency of the public firm is not sufficient to restore the social optimum and contrary to the later, the private leaders are helped by their inability to commit to a given production level. Initiated by the pioneering work of Merrill and Schneider (1966), mixed oligopoly models analyse situations where private profit maximising firms compete with public benevolent ones. They traditionally analyse the effect of concentration and of the order of move in a Cournot-Nash framework (de Fraja and Delbono 1989, Cremer and *al.* 1989, Pal 1998). Usually some assumptions about the cost structure are made to explain that the public firm does not produce the welfare maximising quantity alone. The recent work of Lu and Poddar (2005) is the closest to the present one. They analyse the influence of timing in an investment game between a private and an inefficient public firm in a linear Cournot model where firms choose capacity scales before production. Our approach is different as we use more general assumptions on demand function (log concavity) and strong capacity constraints. These capacity constraints play a crucial role in our model as they do in 'commitment game'. Furthermore we analyse the influence of demand elasticity, cost structure and concentration on the efficiency of the public firm intervention. Our aim is to stress the influence of short term market power on the ability of the public firm to maximize welfare.

Dynamic models of capacity choice have been extensively analysed in the context of duopoly games with a leader and a follower. Spence (1977) and Dixit (1979, 1980) modelled such 'commitment game' to analyse entry deterrence. We use a similar methodology with a benevolent follower. Using a three stage game, Ware (1984) emphasized the commitment value of investment and the relationship with the share of sunk cost. When the follower decides how much capacity to build, he moves along the short term reaction function of the leader.

Sunk costs allow the leader to shape its short term reaction function. The more cost are sunk the more 'constrained' is this shape and the less the entrant will be able to influence the production choice of the incumbent. Here, with a benevolent follower, it is not the constrained part of the reaction function of the incumbent that is embarrassing for the follower. The public follower cannot maximize welfare because of the potential decrease of the private production due to the unconstrained part of the short term reaction function. Therefore, it is the lack of commitment ability which gives the incumbent the possibility to get strictly positive profits.

Using general assumptions, we provide simple conditions characterizing situations where the public firm is able to restore the long term optimum. This condition is an inequality involving the price elasticity of the demand function, the number of firms and the share of sunk cost. The role of these parameters is easily understood given their influence on the short term reaction function of the private oligopoly. For example, for high share of sunk cost the commitment ability of private firms is counter productive as we mentioned above. Some further analysis of the role of these parameters in a linear framework is done, to stress that their influences are not monotonic. Particularly the total capacity decreases for higher concentration.

The article is organized as follows. In section 2, the model is introduced. In section 3, we analyse the case of a unique private firm with the public firm. In section 4, we generalize to the case of an oligopoly of private firms and conclude by the analysis of a linear model.

2. The model

We considered a mixed oligopoly market of a homogenous good. There are $n+1$ firms where firms i , with $i = 1, \dots, n$, are private and firm 0 is public. The inverse demand function is given by $p(q)$ where p is the market price and q the total quantity produced. This function is assumed to be twice differentiable, strictly decreasing and log concave. The log concavity of the inverse demand function means that it is less convex than exponential function.

This property ensures that a firm's profit is quasi concave and that its reaction function is decreasing with a slope above -1 (cf. Vives 1999) (See details in Annex 1). These characteristics of the reaction function are sufficient to existence and uniqueness of Cournot equilibrium. The cost of production is divided into two parts: an irreversible capacity cost and an operating cost. We normalize the long run marginal cost at 1 and the share of capacity cost is noted α .

The output of firm i , $i = 0, \dots, n$, is denoted q_i and its capacity k_i , the firm can produce up to its capacity level $\forall i, q_i \leq k_i$, with marginal cost $1 - \alpha$. We assumed that there exist strictly positive quantities k^* and k^{**} such that $p(k^*) = 1$ and $p(k^{**}) = 1 - \alpha$.

The profits of firms are:

$$\pi_i = (p - (1 - \alpha))q_i - \alpha k_i, i = 0, \dots, n \quad (1)$$

The public firm is benevolent and maximises the social surplus $W(q, k)$:

$$W(q, k) = \int_0^q p(u)du - (1 - \alpha)q - \alpha k \quad (2)$$

It is clearly maximised for $q = k = k^*$. The price elasticity of the demand at the long term optimum k^* is denoted ε , it plays a crucial role in our results. It is defined by:

$$\varepsilon = \frac{p}{p' \cdot k^*} = \frac{1}{p'(k^*) \cdot k^*} \quad (3)$$

We consider the following three-stage game: first the private firms choose capacities then the public firm chooses its capacity, and in the third stage they produce subject to the capacity constraints $q_i \leq k_i, i = 0, \dots, n$.

As we consider a dynamic game with complete information, the more suitable equilibrium concept is the concept of subgame perfect equilibrium (SPE) introduced by Selten (1965, 1975). A notion like SPE assumes common expectations of players' behavior. That is, each player holds a correct conjecture about her opponents' strategy choice. More precisely, strategies are an SPE if whatever the history of the game, strategies are a Nash equilibrium of the subgame.

In our case, it means that whatever the capacity chosen, production quantities are a Nash equilibrium, these quantities are perfectly anticipated by the public firm when choosing its capacity, and this choice is perfectly anticipated by the private firms at the first stage. Consequently, the model resolution is realised by backward induction.

3. A unique private firm

3.1. The production stage

At this stage capacities are fixed. The firms choose production quantities. The private firm maximises its profit (1) and the public one maximises social surplus (2). By considering the private firm's production q_1 as fixed, the public firm produces $q_0(q_1, k_0)$ such that: $p(q_1 + q_0) = 1 - \alpha$ if $q_0 \leq k_0$. Its reaction function is:

$$q_0(q_1, k_0) = \min\{k_0^* - q_1, k_0\} \quad (4)$$

The private firm maximises its profit subject to capacity constraint. In order to describe the private firm's reaction function with capacity constraint, we first introduce the unconstrained reaction function $r(\cdot)$. It is a continuous differentiable function on the set $[0, k_0^*]$. It satisfies the first order condition :

$$p + p'r = 1 - \alpha, \forall q_0 \in [0, k_0^*] \quad (5)$$

Its slope is strictly between 0 and -1 (cf annex 1), and $r(k_0^*) = 0$. The reaction function of the firm with fixed capacity can be described using r :

$$\forall q_0 \in [0, k_0^*], q_1(q_0, k_1) = \min\{k_1, r(q_0)\} = \begin{cases} k_1, & \text{if } q_0 \leq r^{-1}(k_1) \\ r(q_0) & \text{otherwise} \end{cases} \quad (6)$$

There is a unique Nash equilibrium of this game. The total production at this equilibrium is $q^N(k_0, k_1)$. If $k_0 \leq k_0^*$, the capacity of the public firm is binding and the private firm produces. Otherwise the price equals $1 - \alpha$ and the private firm does not produce.

3.2. Public firm choice of capacity

Once the private firm has chosen its capacity k_1 , the public firm has to choose its capacity k_0 . We note $k_0^+(k_1)$ the reaction correspondence of the public firm. The public firm maximizes the social surplus subject to an equilibrium constraint, i.e. the production is the unique Nash equilibrium of the production subgame. Hence we have:

$$k_0^+(k_1) = \arg \max_{k_0} \int_0^{q^N} p(u) du - (1 - \alpha)q^N - \alpha(k_0 + k_1) \quad (7)$$

It is clear that the choice of k_0 could be restricted to the set $[0, k_0^*]$. In that case the capacity constraint of the public firm is binding at the production stage. Therefore the production is $q^N(k_0, k_1) = k_0 + q_1(k_0, k_1)$. The derivative of the social surplus with respect to k_0 is:

$$\frac{dW}{dk_0} = (p - (1 - \alpha)) \frac{\partial q^N}{\partial k_0} - \alpha = \begin{cases} p - 1 & \text{if } 0 < k_0 < r^{-1}(k_1) \\ (p - (1 - \alpha))(1 + r') - \alpha & \text{if } r^{-1}(k_1) < k_0 \leq k_0^* \end{cases} \quad (8)$$

If the public firm chooses a small amount of capacity the private firm will bind its capacity at the production stage and it will not use its short-term market power. In this case the social surplus evolution is usual: a change in k_0 leads to a similar change in q^N and the social surplus is increased by $p - 1$. However, for k_0 greater than

$r^{-1}(k_1)$, the private firm production decreases with respect to k_0 . The impact of this decrease in production on social welfare is represented by the term $(p - (1 - \alpha))r'$.

The long-term social optimum could be reached if the private capacity is binding for $k_0 = k^* - k_1$. In that case $q^N = k^*$ and $p=1$, the price equals the long-term marginal cost. For the social optimum to be reached, the following must hold: $k^* - k_1 \leq r^{-1}(k_1)$. The private firm should produce at full capacity when the public firm invest and produce the long term optimum quantity. Next, conditions under which, such situation is possible are identified. In some cases there is a threshold noted k_A such that $k^* - k_1 \leq r^{-1}(k_1)$ if and only if $k_1 \leq k_A$. This quantity is the solution to the following equation:

$$r(k^* - k_A) = k_A \quad (9)$$

This equation has a solution on the set $[0, k^*]$ if and only if $r(0) \leq k^*$, this solution is unique. This can be easily characterized using the price elasticity ε defined by (\cdot) . As the monopolist's profit is strictly quasi concave, $r(0) \leq k^*$ if and only if $p(k^*) + p'(k^*)k^* \leq 1 - \alpha$ i.e. $p'(k^*)k^* \leq -\alpha$. The threshold k_A exists if and only if $-\varepsilon \leq \frac{1}{\alpha}$. The threshold could be expressed using the elasticity:

$$k_A = -\varepsilon \alpha k^* \quad (10)$$

This threshold could be used to compare the total production, when the private firm's capacity is not binding, with the long-term social optimum:

$$\forall k \in [0, k^{**}], k \leq k^* - k_A \Leftrightarrow k + r(k) \leq k^* \quad (11)$$

We are now able to establish the following lemma which characterized the best response of the public firm.

Lemma 1

If $-\varepsilon \leq \frac{1}{\alpha}$, two situations can be distinguished:

- (i) If $k_1 \leq k_A$: the social surplus is maximized for $k_0^+(k_1) = \{k^* - k_1\}$.
- (ii) If $k_1 > k_A$: the surplus is maximized for $k_0^+(k_1) \subset [r^{-1}(k_1), k^* - k_A]$

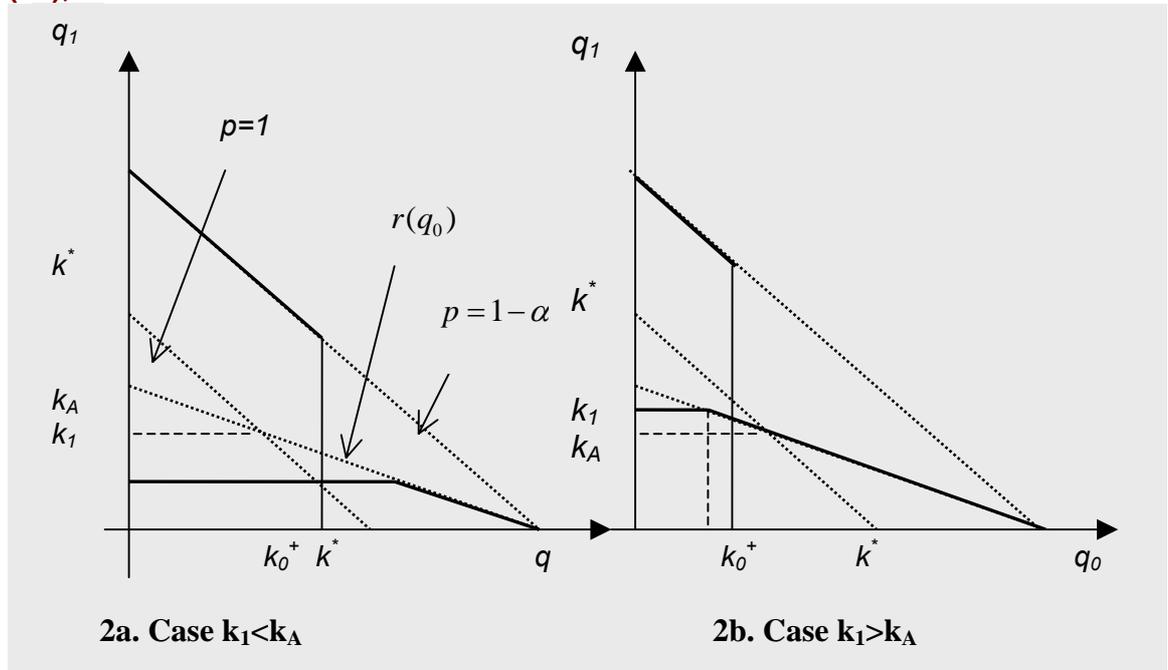
For any quantity in this set price are above the long term marginal cost $p > 1$.

The proof (cf Annex 2) uses the two monotonicity properties of the social welfare with respect to k_0 : (i) it is increasing if the capacity of the private firm is binding and total capacity is less than the social optimum k^* , and (ii) it is decreasing when production is higher than k^* . As the public firm consider the loss of social surplus due to the decrease of private production, its incentive to invest is limited. Even if the public firm can force the price down to the long-term marginal price $p=1$, it is not socially optimal. Although the profit of the public firm is strictly positive in that case, the profit of the private firm could be negative because of unused capacities.

The threshold described by formula (2) is linked to the demand elasticity and the share of the sunk cost α . The less elastic the demand and the smaller the share of sunk costs, the smaller the threshold. It could be easily explained by the links

between the exercise of market power and the price elasticity and variable costs. The incentive for the private firm to restrict production increases with the inelasticity of the demand and with the marginal cost of production.

Figure 2. The public firm's best response k_0^+ in the two situations of lemma 1: if $k_1 < k_A$ (2a), then the long-term optimum can be restored: $k_0^+ = k^* - k_1$. If $k_1 > k_A$ (2b), it cannot be restored



3.3. Private firm choice of capacity

At the first stage of the game the private firm anticipates the reaction of the public firm when choosing its capacity k_1 . As stressed above for a quantity inferior to k_A , the private firm's profit is zero. At the equilibrium of the game, the firm chooses a quantity sufficiently high so that the public firm could not restore the long-term optimum. The price is then greater than the long-term average cost and if the private firm produces at full capacity its profits are strictly positive. The following proposition describes the equilibrium paths of the game according to the value of the different parameters.

Proposition 1

If $-\varepsilon \geq \frac{1}{\alpha}$, there exist an infinity of subgame perfect equilibriums such that $k_1 \in [0, k^*]$, $k_{ISO} = k^* - k_1$ and $p=1$ and profits are nul.

Otherwise, k_A exists and for any $\bar{k} \in \arg \max_{k \geq 0} W(k + r(k), k)$, there exists at least one subgame perfect equilibrium whose equilibrium path is : $k_1 = r(\bar{k}) > k_A$ and $k_0 = \bar{k} < k^* - k_1$ and $q^N = k_1 + k_0$.

If $\arg \max_{k \geq 0} W(k + r(k), k)$ is a singleton, there is a unique subgame perfect equilibrium path.

The proof is in the annex 2. The first case is trivial (cf lemma 1). In the second case the private firm is able to get strictly positive profits thanks to the public firm inability to reach the long term social optimum. All the same, the private firm incentive to underinvest is limited by the public firm intervention.

If the private firm invests less than $r(k^0)$, the public firm's investment and the total quantity produced are increased. So the price decreases and so does the private firm's profit. The private firm is therefore induced to increase its investment. But for greater investment than $r(k^0)$ the private firm's choice has no influence on the public firm's one and the private firm over-invests. Part of her capacity is unused. Although the public firm's profit is strictly positive at equilibrium, the private firm cannot increase its investment to get some of this profit. This is because the firm is unable to credibly commit to a given level of production.

The total investment is superior with the public firm's intervention than without. It can be easily proved by comparing the marginal revenues in both situations. The public firm does not necessarily invest at equilibrium: k^0 could be null for some sets of parameters. Even so, the private firm invests more than a monopolist would.

The uniqueness of a maximizing argument k^0 implies the uniqueness of a subgame perfect equilibrium path. If several arguments maximize $W(k+r(k),k)$, there are several subgame perfect equilibriums with different paths. It is even possible to construct equilibrium where the private firm does not produce at full capacity. It could arise if the public firm's best response switches from one maximizing argument to another.

4. An oligopoly of private firms

We now move on to an oligopoly case. The n private firms choose their capacity $(k_i)_{i=1..n}$ simultaneously at the first stage, followed by the public firm in the second stage. As above the third stage is a production game with fixed capacities, the production of firm $i = 0, \dots, n$ is q_i . We first analyse the production stage followed by the public firm's choice.

Finally we derive some results on the "symmetric" subgame perfect equilibriums of the game. "Symmetric" means that every private firm chooses the same capacity: $k_i = k_j, \forall i, j \in [1, n]$ at the first stage; the public firm's choice may be different. The second and third stages are analysed in the general case of asymmetric capacities. Even if such situations do not arise along the equilibrium path of a symmetric equilibrium they are useful to analyze deviations.

4.1. The production stage

The reaction of an individual private firm is similar to the reaction described by (2). As we want to analyse the reaction of the entire oligopoly to the production choice of the public firm, we introduce a family of functions representing the reaction of an oligopoly. The reaction of a private oligopoly to a fixed production of the public firm is the aggregation of individual reactions to this quantity plus the production of rivals. We note $r(.,m)$ the reaction of m private firms oligopoly when capacity constraints are not binding. Usual results of Cournot oligopoly ensure that these functions are well defined. We have the following relation between the reaction of an individual firm and an oligopoly:

$$m.r\left(\frac{m-1}{m}r(q,m)+q\right)=r(q,m), \forall q \in [0, k^{**}], \forall m \in \square \quad (12)$$

The reaction of the private oligopoly is increasing with respect to the number of firms m , and the derivatives with respect to q is:

$$\frac{\partial r(q,m)}{\partial q} = \frac{m.r'}{1-(m-1)r'} \in]-1, 0[, \forall m \in \square, \forall q \in [0, k^{**}] \quad (13)$$

There exists a unique Nash equilibrium at the production game. The production of firm i at equilibrium is noted as $q_i^N(k_0, k_1, \dots, k_n)$. For $k_0 \in [0, k^{**}]$, the public firm produces at full capacity :

$$q^N(k_0, k_1, \dots, k_n) = k_0 + \sum_{i=1}^n q_i^N(k_0, k_1, \dots, k_n) \quad (14)$$

The private firms' constraints are successively relaxed when the production of the public firm increases. Constraints of private firms with greatest capacities are relaxed first. We formalized in the following lemma.

Lemma 2

For an $n+1$ -tuple $(k_i)_{i=0..n}$, productions at equilibrium verify:

$$\forall i, j \in [1, n], k_i \geq k_j, \text{ then } q_i^N = k_i \Rightarrow q_j^N = k_j$$

The proof is in annex 3. For a fixed $n+1$ -uple of capacities, the total equilibrium production is $q^N = k_0 + k + r(k, m)$, where m is the number of firms whose capacities constraints are not binding, and k is the sum of the $n-m$ other firms' capacities. According to Lemma 2, the firms whose capacity constraint is binding are the firms with the smallest capacities. So, k is the sum of the $n-m$ smallest capacities.

4.2. The second stage: the investment choice of the public firm

The public firm is assumed to be benevolent. It maximizes the social welfare W . The public firm anticipates the third stage equilibrium and therefore maximises W subject to equilibrium constraints. To avoid complications due to the existence of several best responses we assume the uniqueness of the public firm's best response $k_0^+(k_1, \dots, k_n)$.

For a small public capacity, all firms produce at full capacity at equilibrium. The first constraint to be relaxed, as the public firm's capacity increases, is the constraint of the dominant firm i.e. the firm with the largest capacity. If the dominant firm is firm 1, its constraint is relaxed for k_0 greater than the solution to $k_1 = r\left(k_0 + \sum_{i \neq 1} k_i\right)$ which is $k_0 = r^{-1}(k_1) - \sum_{i \neq 1} k_i$. The social optimum could be reached if this constraint is binding for $k_0 = k^* - \sum_i k_i$. The threshold k_A could be used to establish a result similar to lemma 1.

We restrict our attention to situations where the n-tuple of private capacities are in the following set K :

$$K = \left\{ (k_i)_{i=1..n} \middle/ \forall i, k_i \leq r\left(\sum_{j \neq i} k_j\right), \sum_i k_i \leq k^* \right\} \quad (15)$$

The restriction to this set is legitimate (cf Lemma 4) and allows us to simplify the statement and the proof of the following lemma. The choices of private firm's capacities will be in this set at equilibriums which are considered below.

Lemma 3

If $-\varepsilon \leq \frac{1}{\alpha}$, k_A is well defined and for $(k_i)_{i=1..n} \in K$

if $\max(k_i, i = 1..n) \leq k_A$, the long-term optimum can be reached:

$$k_0^+ = k^* - \sum_{i=1}^n k_i, \forall i \in [0, n], q_i^N = k_i, p = 1$$

Otherwise $p > 1$.

This result is similar to lemma 1. One should notice that the threshold does not depend on the number of firms. This threshold should be compared with the dominant firm's capacity and not with total capacity. It is due to the fact that the first constraint to be relaxed is the dominant firm's one. The optimum could be reached if this constraint is still binding for a total production of k^* . Otherwise the short-term market power of the private oligopolists prevents the public firm from reaching the long-term optimum. It seems that increasing the number of firms does not modify the sets of parameters for which the long term optimum could be restored by the public firm. Actually, it does for the set K depends on the number of firm as is stated below.

4.3. The first stage: the choice of capacities $(k_i)_i$ of the private oligopoly.

As mentioned earlier, we analyse symmetric equilibria. To construct equilibria similar to the monopoly case, we define quantities:

$$\bar{k}(n) = \arg \max_k W(k + r(k, n), k) \quad (16)$$

We assumed that these quantities are unique. First, we established that the restriction to the set K is legitimate and that firms produce at full capacity along any subgame perfect equilibrium path.

Lemma 4

At any subgame perfect equilibrium $(k_i)_{i=1..n} \in K$ and all firms produce at full capacity: $\forall i, q_i^N = k_i$.

The proof is in annex 3. The main assumption used to establish this result is the uniqueness of the public firm best response. A private firm who does not produce at full capacity could decrease its capacity without modifying the public firm's choice; such a change is beneficial to the firm. With this result, lemma 3 could be used to analyse equilibrium. Although the threshold does not depend on the number of firms in the oligopoly, the two quantities are linked via the set K . When the firms are numerous any symmetric n-tuple in the set K satisfies the condition of the lemma 3. The critical number of firms is determined by the price elasticity of the demand at k^* and the share of the irreversible cost in the total cost. These results constitute the following lemma.

Lemma 5

The following equivalence is verified:

$$-\varepsilon \geq \frac{1}{n} \frac{1}{\alpha} \Leftrightarrow r(0, n) \geq k^* \Leftrightarrow \forall (k, \dots, k) \in K, k \leq k_A$$

The proof is in annex 3. This lemma states that any symmetric elements of K satisfied the conditions of lemma 3 if and only if the inequality linking elasticity, concentration and cost is satisfied. These lemmas pave the way to the following generalization of proposition 1.

Proposition 2

If $-\varepsilon < \frac{1}{n} \frac{1}{\alpha}$ then there is a subgame perfect equilibrium such that along the equilibrium path:

$$k_i = \frac{r(\bar{k}(n), n)}{n}, i = 1..n \text{ and } k_0 = \bar{k}(n), p > 1 \text{ and } \pi_i > 0, i = 0..n$$

Otherwise, the set of the paths of symmetric subgame perfect equilibriums is:

$$\left\{ (k_i = k)_{i=1..n}, k_0 = k^* - nk \middle/ k \in \left[\frac{k^* - k_A}{n-1}, \frac{k^*}{n} \right] \right\}$$

and $p=1$ and $\pi_i = 0, i = 1..n$.

Proposition 2 generalises proposition 1. The proof (cf annex 3) is longer because of the complexity of the short-term reaction of the oligopoly. However, the logic is the same. If an individual firm deviates from a symmetric equilibrium by increasing its capacity, this does not modify the public firm's choice and some of the deviator's capacity would be unused. If a firm deviates by decreasing its capacity, the public firm must increase capacity and consequently, both price and profits decrease. The existence of a suboptimal symmetric equilibrium depends on the number of firms, the price elasticity and the share of irreversible cost in total cost α . Such an

equilibrium exists if and only if $-\varepsilon < \frac{1}{n} \frac{1}{\alpha}$. The less elastic the demand is the more numerous the firms should be so that the public firm can restore the long-term social

optimum. A decrease of elasticity increases the incentive for firms to limit production on the short-term and the difficulty for the public firm to restore the long-term optimum. Similarly an increase in the variable cost decreases the short-term production. The less the ratio of sunk costs over total costs, the more numerous the firms should be for the long-term optimum to be restored at equilibrium.

We focused on symmetric equilibria because of their analytical tractability. Asymmetric equilibria should exist and firms may be able to get strictly positive profits along such equilibria even in the second case of proposition 2.

This proposition explains in which situation the long term optimum could be reach. It does not give any results on the distance to this optimum when the condition is not satisfied. Actually, the evolution of the social welfare could be opposite to the evolution of the inequality. For example, when concentration increases the proposition states that the set of parameters such that the optimum is reached increases, but for 'suboptimal' parameters the situation may worsen. We discuss it below.

4.4. Some comparative static results

A comparative static analysis is difficult in the general case. The monotonicity of the total investment with respect to the number of firms could not be easily deduced with our assumptions. When the concentration is sufficiently low, any symmetric equilibrium leads to the long-term optimum. Yet for a high concentration i.e. low number of firms, the total investment and production are $\bar{k}(n) + r(\bar{k}(n), n)$, for which the evolution with respect to n is not evident. Either $\bar{k}(n) = 0$ or it satisfies the first order condition: $(p - c) \left(1 + \frac{\partial r}{\partial q} \right) = \alpha$. Hence, the monotonicity of $\bar{k}(n)$ according to n is the same as the monotonicity of the left side of the equation. And this is decreasing according to n if $\frac{\partial r}{\partial q}$ is. The change of production of a n -firm oligopoly described by (13) should be greater the more numerous are the firm, which seems to be a quite natural assumption given that it is maximized when firms are competitive. It is then possible that the production of public firm and the total investment decrease with respect to the number of private firm for high concentrations. It is the case with a linear demand as shown in the following example.

In the linear case, consider the following demand function: $p(q) = a - bq$. Then, the reaction function of an 'unconstrained' oligopoly is:

$$r(q, n) = \frac{n}{n+1} \left(\frac{a - (1 - \alpha)}{b} - q \right)$$

The capacities chosen along a symmetric equilibrium path are:

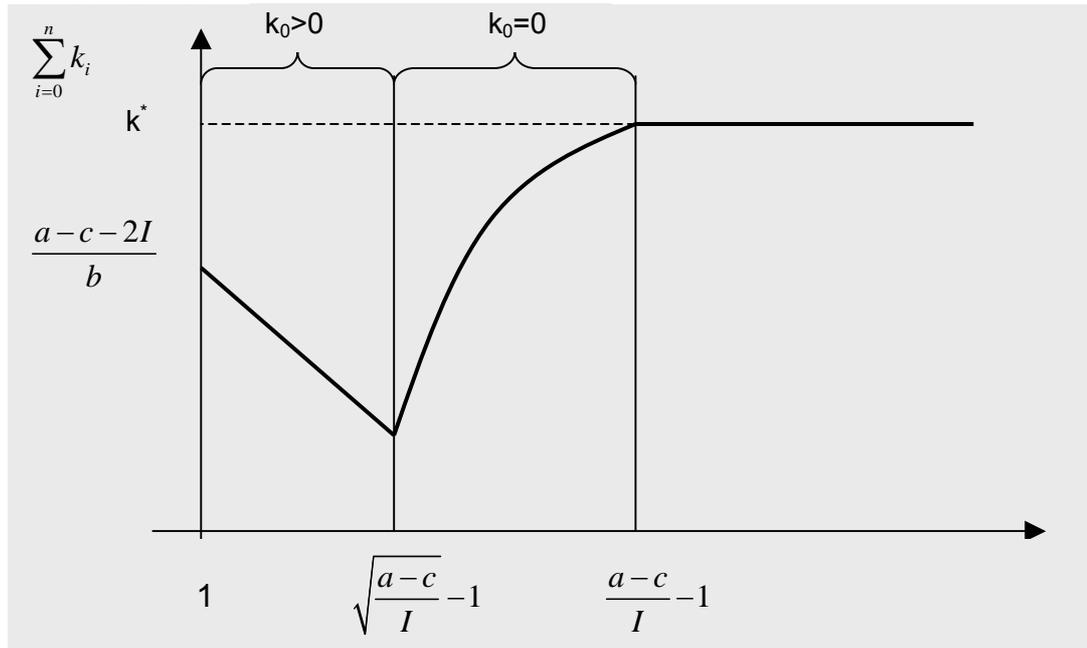
$$\text{If } n+1 \leq \sqrt{\frac{a - (1 - \alpha)}{\alpha}},$$

$$k_1 = \dots = k_n = \frac{1}{n} r(k^0(n), n) = \frac{(n+1)\alpha}{b}, k^0(n) = \frac{1}{b} (a - (1 - \alpha) - (n+1)^2 \alpha),$$

if $\sqrt{\frac{a-(1-\alpha)}{\alpha}} \leq n+1 \leq \frac{a-(1-\alpha)}{\alpha}$, $k_1 = \dots = k_n = \frac{1}{n+1} \frac{a-(1-\alpha)}{b}$, $k^0(n) = 0$,
 and if $\frac{a-(1-\alpha)}{\alpha} \leq n+1$, the total investment and production are k^* .

The total investment with respect to n is represented in Figure 3.

Figure 3. Total capacity and production with respect to the concentration for a linear demand

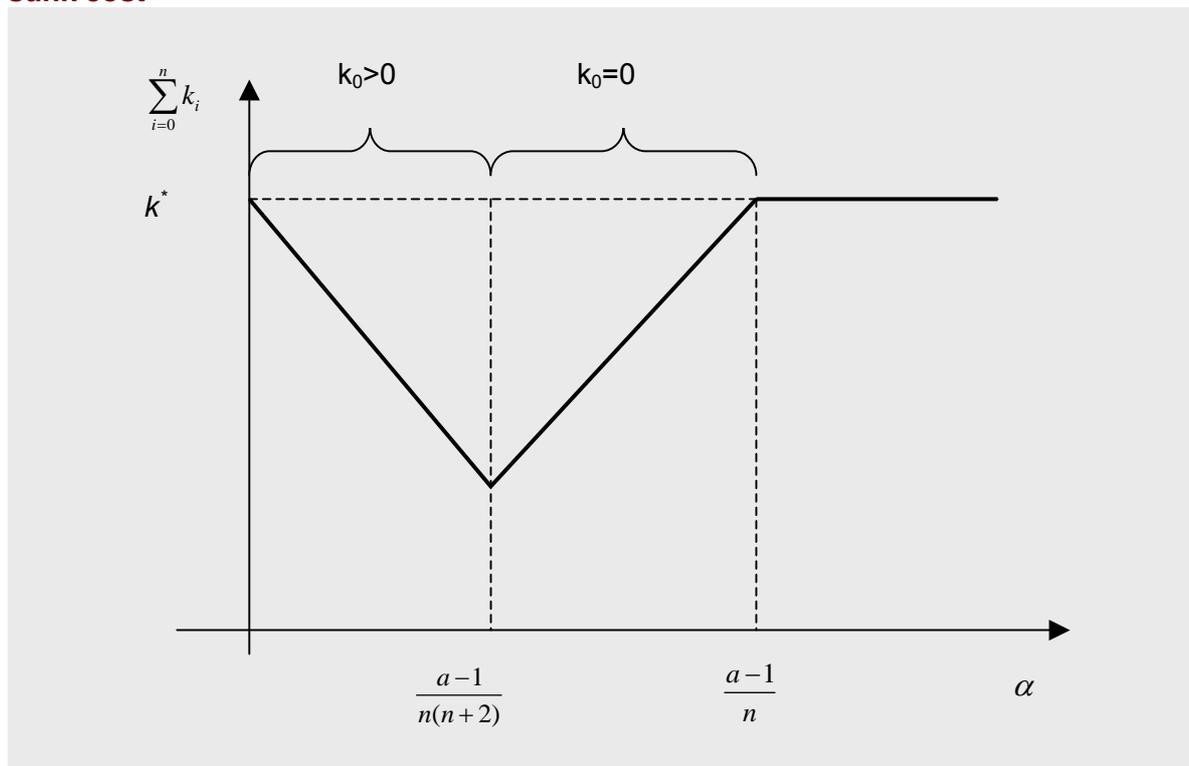


There are two drivers of the evolution of investment when the number of private firms increases. First, the public firm's incentive to invest decreases because of the short term reaction of the private oligopoly and second, competition between private firms increases. The first driver dominates for high concentration and the total investment decreases, up to the point where the public firm no longer invests. From this point, competition among firms ensures that the total investment increases according to the number of private firms.

The same analyse could be done according to the share of sunk cost. Proposition 2 states that for $\alpha \geq -\varepsilon \frac{1}{n}$, there are subgame perfect equilibrium such that the long

term optimum is reached. But for suboptimal situations total investment is not increasing according to α . Figure 4 depicts the linear case, calculations are straightforward given the above results. For low share of sunk cost an increase of sunk cost increases the social loss due to the replacement of production by new capacities. The commitment value of private firms' capacities increases and the public firm's ability to restore the social optimum decreases. The total investment decreases up to the point where the public firm no longer invests in generating capacities. From this point competition between private firms ensures that investment increases according to the share of sunk cost.

Figure 4. Total capacity and production evolution with respect to the share of sunk cost



5. Conclusion

Given the oligopolistic structure of electricity markets, there is a worry that firms might use their long-term market power and under-invest in generation capacity. We have tested here the efficiency of public firm's intervention to compensate this under-investment. The public firm aims to re-establish the social optimum by completing the investment of the private firms. In a conventional model of Cournot oligopoly with joint decisions of capacity investment and production, such a solution is efficient. In our model, we dissociate the step of investment and the step of generation. In this setting, the short-term market power changes the social efficiency of the instrument.

Contrary to usual commitment games where incumbents' advantage is linked to their ability to commit to a given production here, incumbents' inability to commit is the key force that allows them to get strictly positive profits. As the public firm anticipates the influence of its capacity choice on the short-term total production, the increase of the social surplus due to the supplement of production is partly compensated by the decrease of the production of the private oligopoly, induced by the exerting of market power. In the long-term the private firms remain able to get strictly positive profits and to keep the sector in a sub-optimal position. However, this situation is better than the case without a public firm intervention even if the public firm does not invest. The degree of demand elasticity and the level of concentration determine the possibility to move nearer the optimum. For elastic demand functions and (or) decentralised sectors, the optimum could be reached. On the other hand we demonstrate that, if the concentration is high, increasing the number of firms could imply greater difficulty for the public firm to move closer to the optimum.

These results are relevant indications for public policies to correct the long term market power exercise consisting in restricted investment in generation capacity on electricity markets. It concerns restriction in investment in base load, semi-base load and peaking units for keeping price at higher level than optimal one, and more specifically the capacity deficit in reserve units resulting from combination of producers' risk aversion and market power. Investment in capacity by a benevolent and competitive player corrects partly the situation, but with the private firms keeping means to make monopolistic profits.

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Mathematical Appendix

Annex 1

1.1. Log concavity and best reaction function

The log concavity of the demand function is a common assumption. As $\log p$ is concave, the function p satisfies $p''p - (p')^2 \leq 0$ as long as $p > 0$. Then, if x is a solution to the first order equation: $p'(q+x)x + p(q+x) = 1 - \alpha$ for $q \in [0, k^{**}]$.

Hence, $p'' \leq \frac{(p')^2}{p} \leq \frac{(p')^2}{p'x}$ so $p' + p''x \leq 0$, i.e. the cross derivative of the profit is negative, and, $2p' + p''x \leq 0$ the second order condition is satisfied. Hence, the profit function is strictly quasi concave. It ensures the existence and uniqueness of the best response of the private firm.

The slope of the function r is given by: $\frac{p' + p''r}{2p' + p''r}$ which is clearly in the set $]-1, 0[$.

1.2. Existence and value of the threshold k_A

As the slope of r' is strictly between 0 and -1 if the solution of the equation $r(k^* - k) = k$ exists, it is unique.

As $r(k^*) > 0$, the solution is in the set $[0, k^*]$ if and only if $r(0) \leq k^*$, and given quasi concavity of the profit it is equivalent to $p(k^*) + p'(k^*)k^* - (1 - \alpha) \leq 0$ i.e.

$$\frac{p'k^*}{p} \leq -\alpha \text{ or } -\varepsilon \leq \frac{1}{\alpha}$$

The threshold could be written using the price elasticity.

From the first order condition: $p(k^*) + p'(k^*)k_A = 1 - \alpha$, so, $k_A = \frac{-\alpha}{p'(k^*)} = -\varepsilon \alpha k^*$.

Annex 2

2.1. Proof of Lemma 1

The result is due to the following monotonicity properties of the social surplus:

(i) $W(q^N(k_0, k_1), k_1 + k_0)$ is increasing with respect to k_0 if the capacity of the private firm is binding at the production stage: $q_1 = k_1$, and the total production is less than the long-term optimum: $q^N = k_1 + k_{ISO} \leq k^*$.

(ii) W is decreasing if the production is greater than the long-term optimum: $q^N(k_0, k_1) \geq k^*$.

For intermediary situations the monotonicity of W is not clear.

If $k_1 \leq k_A$, the monopolist's capacity is binding for $k_0 = k^* - k_1$. So W is maximized at $k^* - k_1$.

If $k_1 > k_A$, the social surplus is increasing for $k_0 \leq r^{-1}(k_1)$ and decreasing for $q^N \geq k^*$ i.e. $k_0 \geq k^* - k_A$. Then W is maximized for k_0 in the set $[r^{-1}(k_1), k^* - k_A]$.

2.2. Proof of proposition 1

The first case is trivial; in that case whatever the private firm's choice, the public firm is able to reach the long-term optimum.

In the second case, the threshold k_A is well defined.

Let $\bar{k} \in \arg \max W(k + r(k), k)$.

We do not precisely describe the strategy of the public firm as we need only some properties.

For $k_1 \geq r(\bar{k})$, \bar{k} maximizes the social surplus:

$$\forall k_0 \geq r^{-1}(k_1), W = W(k_0 + r(k_0), k_0 + k_1) = W(k_0 + r(k_0), k_0) - Ik_1$$

Hence the strategy $k_0(k_1) = \bar{k}$ is a best response.

For $k_1 \in [k_A, r(k^0)]$ we only need to know that $k_0^+(k_1) \subset [r^{-1}(k_1), k^* - k_A]$ (cf lemma 1).

Next, we state that $k_1 = r(k^0)$ is the optimal choice for the private firm if the public firm's strategy is $k_0(k_1) = \bar{k}, \forall k_1 \in [r(\bar{k}), r(0)]$.

For $k_1 \in [r(\bar{k}), r(0)]$, it is clear that the firm's profit is decreasing.

For $k_1 \in [k_A, r(\bar{k})]$ we know that the public capacity is above $r^{-1}(k_1)$ so $\bar{k} \leq r^{-1}(k_1) \leq k_0$. The production q^N is increasing with respect to the public capacity so $k_0 + r(k_0) \geq \bar{k} + r(\bar{k})$ and the price is reduced:

$$0 \leq p(k_0 + r(k_0)) - 1 \leq p(\bar{k} + r(\bar{k})) - 1$$

Finally by multiplying by the production and capacity, we get:

$$(p - c) \cdot q_1(k_0, k_1) - \alpha \cdot k_1 \leq (p - c) \cdot r(k^0) - \alpha \cdot r(k^0)$$

The profit of the private firm is greater for $k_1 = r(\bar{k})$ and the private firm's profit is strictly positive for $k_1 = r(\bar{k})$. Q.E.D.

Annex 3

3.1. Proof of Lemma 2

Let $i, j, k_i \geq k_j$, we assume that $q_i^N = k_i$. Using the first order condition:

$p + p'k_i \geq 1 - \alpha$, hence $p + p'k_j \geq 1 - \alpha$ and $q_j^N = k_j$. Q.E.D.

3.2. Proof of Lemma 3

We assume that $k_1 = \max\{k_i, i = 1..n\}$.

If $k_1 \leq k_A$, then for $k_0 = k^* - \sum_i k_i$, $r\left(k_0 + \sum_{i=2}^n k_i\right) = r(k^* - k_1) \geq k_1$.

It implies that at the production stage the capacity constraint of all firms is binding. The production is $q^N = k^*$ and W is maximised.

If $k_1 \geq k_A$, for $0 \leq k_{ISO} \leq r^{-1}(k_1) - \sum_{i=2}^n k_i$, all firms produce at full capacity and $q^N = \sum_i k_i \leq k^*$. Therefore W is increasing with respect to k_0 . For greater k_0 , the

derivative of W is $\frac{dW}{dk_0} = (p - c) \frac{\partial q^N}{\partial k_0} - I < (p - c) - I$ which is negative if $q^N = k^*$.

Q.E.D.

3.3. Proof of Lemma 4

The assumption of uniqueness of the public firm's best response is the key assumption. We first state that firms produce at full capacity. If some firms do not produce at full capacity, we know that the dominant firm is among one of them. In that case a slight decrease of the dominant firm capacity does not influence the public firm's choice because of uniqueness of the public firm's best response. Such a change would increase profits of the dominant firm by diminishing its investment in unused capacities. The deviation is then profitable.

It is now straightforward to establish the result. The set K is defined by 2 conditions:

- (i) $\forall i \geq 1, k_i \leq r\left(\sum_{j \neq i, 0} k_j\right)$ is necessary so that firms produce at full capacity,
- (ii) $\sum_{i=1} k_i \leq k^*$ is necessary, so that firms get positive profits. QED

3.4. Proof of Lemma 5

By definition $n.r\left(\frac{n-1}{n}r(0, n)\right) = r(0, n)$.

As $r(\cdot)$ is decreasing $r(0, n) < k^* \Leftrightarrow r\left(\frac{n-1}{n}k^*\right) < \frac{k^*}{n}$ and using the strict quasi

concavity of the profit : $r\left(\frac{n-1}{n}k^*\right) < \frac{k^*}{n} \Leftrightarrow p(k^*) + p'(k^*)\frac{k^*}{n} < 1 - \alpha$ which is

equivalent to $-\varepsilon < \frac{1}{n} \frac{1}{\alpha}$. The first equivalence is proved.

And the second equivalence is straightforward:

$$r(0, n) \geq k^* \Leftrightarrow r\left(k^* - \frac{k^*}{n}\right) = r\left(\frac{n-1}{n}k^*\right) \geq \frac{k^*}{n} \Leftrightarrow \frac{k^*}{n} \leq k_A. \text{QED}$$

3.5. Proof of proposition 2

If $-\varepsilon < \frac{1}{n} \frac{c+I}{I}$ then $r(0, n) < k^*$ (lemma 5).

Hence $k^0(n) + r(k^0(n), n) < k^*$ and $\frac{r(k^0(n), n)}{n} \geq k_A$.

We assume that $k_2 = \dots = k_n = \frac{r(k^0(n), n)}{n}$ and state that $k_1 = k_2$ maximizes firm 1's profit. It depends on the public firm's strategy $k_0^+(k_1, \dots, k_n)$.

It is clear that $k_0^+(k_2, k_2, \dots, k_2) = k^0(n)$.

For $k_1 \geq k_2$:

The quantity produced by the oligopoly with respect to k_0 is noted $q(\cdot)$:

$$q(k_0) = \begin{cases} k_1 + (n-1)k_2 & \text{if } 0 \leq k_0 \leq r^{-1}(k_1) - (n-1)k_2 \\ (n-1)k_2 + r((n-1)k_2 + k_0) & \text{if } r^{-1}(k_1) - (n-1)k_2 \leq k_0 \leq k^0(n) \\ r(k_0, n) & \text{if } k^0(n) \leq k_0 \end{cases}$$

Firm 1's capacity constraint is relaxed first. It is relaxed for $k_0 = r^{-1}(k_1) - (n-1)k_2$ which is smaller than $k^0(n)$. The constraints of the other firms are relaxed for $k_0 \geq k^0(n)$.

The production of a constrained oligopoly is less than the production of an unconstrained one : $\forall k_0, q(k_0) \leq r(k_0, n)$. The social surplus is increasing with respect to quantity for $p > c$ so :

$$W\left(q(k_0), \sum_i k_i\right) \leq W\left(k_0 + r(k_0, n), \sum_i k_i\right)$$

And the second term is maximized when $k_0 = k^0(n)$. Thus, $k_0^+ = k^0(n)$ and firm 1 overinvests.

For $k_1 \leq k_2$:

The social surplus is increasing as long as all constraints are binding.

The public firm's best response satisfies $k_0^+ \geq r^{-1}(k_2) - (n-2)k_2 - k_1$ and the production is greater than $k^0(n) + r(k^0(n), n)$. The price is therefore less than

$p(k^0(n) + r(k^0(n), n))$ obtained for $k_1 = k_2$ and the profit of firm 1 is less than the profit obtained for $k_1 = k_2$.

Hence $k_1 = k_2$ is the best response of firm 1.

If $-\varepsilon < \frac{1}{n} \frac{c+I}{I}$, then all symmetric subgame perfect equilibria are cases of lemma 3.

The set $\left\{ (k_i = k)_{i=1..n}, k_0 = k^* - nk / k \in \left[\frac{k^* - k_A}{n-1}, \frac{k^*}{n} \right] \right\}$ describes all the paths:

For $k_1 = \dots = k_n \in K$ at equilibrium, a profitable deviation for firm 1 should be for a quantity greater than k_A and less than $k^* - (n-1)k_2$ which is possible if and only if

$$k_2 \leq \frac{k^* - k_A}{n-1}. \text{ QED}$$